COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science

Sample Mathematics Assessment for the Online AI executive Certificate

1. What is an example of two sets A, B that satisfv
   $|A \cap B| = |A - B| = |B - A| = 1$?
   $|A|$ is the cardinality of A.
   I. $A = \{\emptyset, \{\{\emptyset\}, 2\}\}, B = \{\{\emptyset\}, 2\}$
   II. $A = \{\emptyset, \{\emptyset\}, 2\}, B = \{\emptyset, 2\}$
   III. $A = \{\{\emptyset\}, \{\emptyset, 2\}\}, B = \{\emptyset, 2\}$
   IV. $A = \{\emptyset, \{\emptyset, 2\}\}, B = \{\{\emptyset, 2\}\}$
   V. $A = \{\emptyset, \{\emptyset, 2\}\}, B = \{\{\emptyset\}, \{\emptyset, 2\}\}$

2. Let $x_0 = 1$, $x_k = \frac{1}{3}x_{k-1}$. Calculate $\sum_{k \geq 0} x_k$

   I. $\frac{1}{2}$

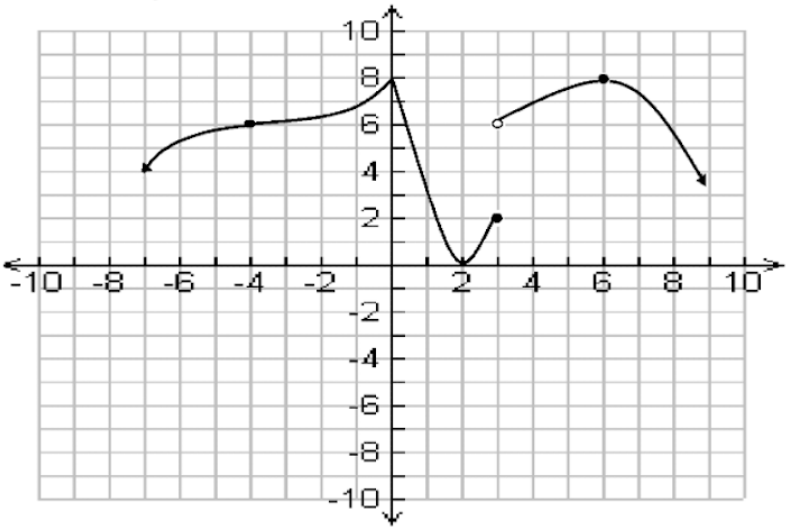
   II. $\frac{3}{2}$

   III. $\frac{5}{3}$

   IV. 2

   V. ∞

3. The function $f: \mathbb{R} \to \mathbb{R}$ that has the following graph.



a) Find the critical points of f .

b) Find the intervals where the second derivative, f'', is positive.

I. $(-\infty, -4), (3, \infty)$
II. $(-\infty, 0), (2,3), (3,6)$
III. $(-4,3)$
IV. (-4,0), (0,3)
V. $(-\infty, \infty)$

4. Let $f(x,y) = x^2 + xye^y - e^{-y^2}$.

Find $\frac{df}{dy}(x,y)$.

I. $2x + xe^y - e^{-y^2}$

II. $x(y+1)e^y + 2ye^{-y^2}$

III. $xe^y + 2ye^{-y^2}$

IV. $xe^y - e^{-y^2}$

V. $xe^y + xye^y - e^{-y^2}$

5. Let $v_1 = (3\ -2\ 1)$, $v_2 = (-1\ 1\ 5)$ *and* $v_3 = (1\ 1\ -1)$.

Does the set $A = \{v_1, v_2, v_3\}$ form a orthogonal basis in R^3?

I. A is an orthogonal basis of R^3.

II. A is a basis of R^3 but not an orthogonal set of vectors.

III. A is an orthogonal set of vectors but not a basis in R^3.

IV. A is not a basis of R^3 and not an orthogonal set of vectors.

V. None of the above.

6. Let $A: \mathbb{R}^3 \to \mathbb{R}^3$ a linear transformation. $A = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 3 & 8 \\ 1 & 0 & 1 \end{bmatrix}$

Find the eigenvalues of A

I. $\lambda_1 = 3, \quad \lambda_2 = 2 + \sqrt{2}, \quad \lambda_3 = 2 - \sqrt{2}$

II. $\lambda_1 = -3, \quad \lambda_2 = -2 + \sqrt{2}, \quad \lambda_3 = -2 - \sqrt{2}$

III. $\lambda_1 = 3, \quad \lambda_2 = 2 + \sqrt{3}, \quad \lambda_3 = 2 - \sqrt{3}$

IV. $\lambda_1 = 3, \quad \lambda_2 = 3 + \sqrt{2}, \quad \lambda_3 = 3 - \sqrt{2}$

V. $\lambda_1 = -3, \quad \lambda_2 = 3 + \sqrt{3}, \quad \lambda_3 = 3 - \sqrt{3}$

7. Assume that unvaccinated people are 7 times more likely to get COVID-19 than vaccinated people and that 60% of people are vaccinated. Assume that there is a 1% chance that a random individual will get COVID-19 in October 2021.

a. Calculate the probability that a vaccinated individual gets COVID-19 in October.

I. I0.05%

II. 0.1%

I. 0.2%

II. 0.3%

III. 0.5%

7b. (refer to first part of question 7)

Calculate the probability that an individual is vaccinated and gets COVID-19 in October.

I. 0.08%

II. 0.10%

III. 0.12%

IV. 0.18%

V. 0.30%

7c. (refer to first part of question 7)
Calculate the probability that an individual gets COVID-19 in October was vaccinated.

I. 0.6%
II. 0.10%
III. 0.12%
IV. 0.14%
V. 0.18%

8. Assume the average weight of an adult male is 172 pounds with a standard deviation of 28 pounds. Assume that the weight of adult males is normally distributed. Find the probability that a randomly selected adult male weighs more than 200 pounds.

I. 0.32
II. 0.16
III. 0.08
IV. 0.05
V. 0.025